Journal of Engineering Physics and Thermophysics, Vol. 74, No. 5, 2001

GENERATION OF HIGH-FREQUENCY HARMONICS IN THE CASE OF LARGE-AMPLITUDE VIBRATIONS OF A GAS IN A CLOSED TUBE

A. L. Tukmakov UDC 534.222

Based on the model of a viscous compressible heat-conducting gas, the author investigates numerically the vibrations of a gas column in a closed tube in excitation by a flat piston that moves according to a harmonic law with an amplitude comparable to the resonator length. Consideration is given to the process of transformation of the mode and frequency of vibrations of the gas column for a fixed excitation frequency; the transformation is accompanied by a change of the type of resonance and is related to the increase in the eigenfrequency of the system due to the increase in the average temperature of the gas.

In the case of longitudinal vibrations of a gas in an acoustic resonator of the closed-tube type near the resonant frequencies $\omega_{mn} = n\pi a_0/(ml)$ (m, n = 1, 2, 3, ...), one observes the establishment of discontinuous or nearly discontinuous vibrations. The value $m = 1$ corresponds to the natural frequencies (eigenfrequencies) of the gas column (n = 1, 2, 3, ...). The values $n = 1$ and $m > 1$ correspond to the subharmonic frequencies at which nonlinear resonances are realized. The resonant vibrations of the gas for m = 1 and 2 were investigated in numerous experimental and theoretical works reviewed in [1]. Subharmonic resonance for $n = 1$ and $m = 3$ was detected experimentally for the first time comparatively recently [2]. A theoretical description of the experimental results of [2] within the framework of the model of a compressible heat-conducting gas is contained in [3]. The known theoretical and experimental works [1] were carried out under the assumption that the amplitudes of vibrations of the exciting element in a resonant tube are substantially smaller than the tube length. This restriction caused the eigenfrequencies of the system to change with time in a comparatively small range, as a result of which the gasdynamic processes were considered near the initial state of the system. In the present work, we investigate the behavior of the resonant system with amplitudes of piston vibrations comparable to the system's length.

To describe the motion of a gas in the tube, we used the system of Navier–Stokes equations for a compressible heat-conducting gas [4, 5] written in a cylindrical coordinate system:

$$\mathbf{q}_t + \mathbf{F}_x + \mathbf{G}_y = \mathbf{H}\,,$$

$$\mathbf{q} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ E \end{bmatrix}; \quad \mathbf{F} = \begin{bmatrix} \rho u \\ \rho u^2 + p - \tau_{xx} \\ \rho uv - \tau_{xy} \\ (E + p - \tau_{xx})\, u - \tau_{xy} v + Q_x \end{bmatrix}; \quad \mathbf{G} = \begin{bmatrix} \rho v \\ \rho uv - \tau_{xy} \\ \rho v^2 + p - \tau_{yy} \\ (E + p - \tau_{yy})\, v - \tau_{xy} u + Q_y \end{bmatrix};$$

Institute of Mechanics and Mechanical Engineering, Kazan Scientific Center, Russian Academy of Sciences, Kazan, Russia; email: tukmakov@sci.kcn.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 74, No. 5, pp. 39–43, September–October, 2001. Original article submitted April 21, 2000; revision submitted December 19, 2000.

$$\mathbf{H}=\begin{bmatrix} -\rho v/y \\ (-\rho uv+\tau_{xy})/y \\ (-\rho v^2+\tau_{yy})/y \\ \left(-v\,(E+p-\tau_{yy})+k\dfrac{\partial T}{\partial y}+u\tau_{xy}\right)/y \end{bmatrix};\quad p=(\gamma-1)\,(E-0.5\rho\,(u^2+v^2))\,;$$

$$Q_x=-\left(k+\frac{c_p\mu_t}{\mathrm{Pr}_t}\right)\frac{\partial T}{\partial x};\quad Q_y=-\left(k+\frac{c_p\mu_t}{\mathrm{Pr}_t}\right)\frac{\partial T}{\partial y};\quad D=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{v}{y}\,;$$

$$\tau_{xx}=(\mu+\mu_t)\left(2\frac{\partial u}{\partial x}-\frac{2}{3}D\right);\quad \tau_{yy}=(\mu+\mu_t)\left(2\frac{\partial v}{\partial y}-\frac{2}{3}D\right);\quad \tau_{xy}=(\mu+\mu_t)\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right). \tag{1}$$

System (1) in the region with variable boundaries was solved by introduction of generalized coordinates [4, 6] $\xi = \xi(x, y, t)$, $\eta = \eta(x, y, t)$, $\tau = t$. In new variables, system (1) has the form

$$\mathbf{q}_t^*+\mathbf{F}_\xi^*+\mathbf{G}_\eta^*=\mathbf{H}^*\,, \tag{2}$$

$$\mathbf{q}^*=\frac{1}{J}\begin{bmatrix}\rho\\ \rho u\\ \rho v\\ E\end{bmatrix};\quad \mathbf{H}^*=\frac{1}{J}\begin{bmatrix} -\rho v/y \\ (-\rho uv+\tau_{xy})/y \\ (-\rho v^2+\tau_{yy})/y \\ \left(-v\,(E+p-\tau_{yy})+k\dfrac{\partial T}{\partial y}+u\tau_{xy}\right)/y \end{bmatrix};$$

$$\mathbf{F}^*=\frac{1}{J}\begin{bmatrix} \xi_t\,\rho+\xi_x\,\rho u+\xi_y\,\rho v \\ \xi_t\,\rho u+\xi_x\,(\rho u^2+p-\tau_{xx})+\xi_y\,(\rho uv-\tau_{xy}) \\ \xi_t\,\rho v+\xi_x\,(\rho uv-\tau_{xy})+\xi_y\,(\rho v^2+p-\tau_{yy}) \\ \xi_t\,E+\xi_x\,((E+p-\tau_{xx})\,u-\tau_{xy}v+Q_x)+\xi_y\,((E+p-\tau_{yy})\,v-\tau_{xy}u+Q_y) \end{bmatrix};$$

$$\mathbf{G}^*=\frac{1}{J}\begin{bmatrix} \eta_t\,\rho+\eta_x\,\rho u+\eta_y\,\rho v \\ \eta_t\,\rho u+\eta_x\,(\rho u^2+p-\tau_{xx})+\eta_y\,(\rho uv-\tau_{xy}) \\ \eta_t\,\rho v+\eta_x\,(\rho uv-\tau_{xy})+\eta_y\,(\rho v^2+p-\tau_{yy}) \\ \eta_t\,E+\eta_x\,((E+p-\tau_{xx})\,u-\tau_{xy}v+Q_x)+\eta_y\,((E+p-\tau_{yy})\,v-\tau_{xy}u+Q_y) \end{bmatrix};$$

$$J=\begin{bmatrix}\xi_x & \xi_y & \xi_t\\ \eta_x & \eta_y & \eta_t\\ 0 & 0 & 1\end{bmatrix};\quad \xi_t=-x_t\,\xi_x-y_t\,\xi_y\,;\quad \eta_t=-x_t\,\eta_x-y_t\,\eta_y\,.$$

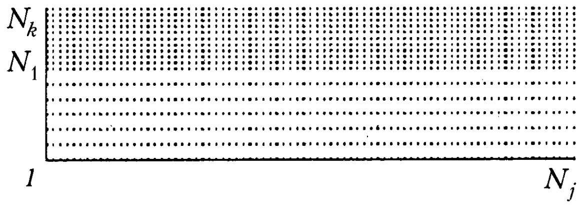

Fig. 1. Fragment of the finite-difference grid.

To solve system (2), we used the explicit method of McCormack of second order of accuracy with time splitting and a scheme of correction of fluxes [4]. On a uniform grid, McCormack's scheme contains two steps that carry out the transition to the next time layer: the predictor step and the corrector step

$$q^{0}_{j,k} = q^{n}_{j,k} - \frac{\Delta t}{\Delta \xi}\,[F^{n}_{j+1,k} - F^{n}_{j,k}] - \frac{\Delta t}{\Delta \eta}\,[G^{n}_{j,k+1} - G^{n}_{j,k}] + \Delta t H^{n}_{j,k}\,;$$

$$q^{n+1}_{j,k} = 0.5\,(q^{n}_{j,k} + q^{0}_{j,k}) - 0.5\,\frac{\Delta t}{\Delta \xi}\,[F^{0}_{j,k} - F^{0}_{j-1,k}] - 0.5\,\frac{\Delta t}{\Delta \eta}\,[G^{0}_{j,k} - G^{0}_{j,k-1}] + 0.5\,\Delta t H^{0}_{j,k}\,.$$

Each three-dimensional group **F** and **G** at the predictor and corrector steps is approximated by unilateral finite-difference operators, Thus, for example, at the predictor step the derivatives with respect to ξ that appear in $F^{n}_{j+1,k}$ and $F^{n}_{j,k}$ are approximated by the left-hand difference schemes of first order of accuracy, while at the corrector step they are approximated by the right-hand schemes; the derivatives with respect to η are approximated by the central schemes of second order of accuracy. The derivatives with respect to η that appear in $G^{n}_{j,k+1}$ and $G^{n}_{j,k}$ are approximated by the left-hand difference schemes of first order of accuracy, while the derivatives with respect to ξ are approximated by the central schemes. The derivatives with respect to ξ and η appearing in **H** at the predictor and corrector steps are approximated by the central difference schemes of second order of accuracy. In calculations on a nonuniform grid with bunching near the lateral surface, we introduced the scheme of time splitting [4, 5]. On the finite-difference scheme (whose fragment is shown in Fig. 1) in the physical region (x, y), the step on the x axis is uniform. We prescribed N_k nodes along the y axis. The first N_1 nodes formed cells with a fixed step Δy $(0 \le y \le rd/2)$. The nodes from N_1 to N_k $(rd/2 < y \le d/2)$ formed cells with a finer step Δy_1. The parameter r prescribed the boundary of the region with a fine step. The region (ξ, η) represented a unit square with uniform subdivision along the ξ and η axes.

The scheme of splitting for the region in question was prescribed by a symmetric sequence of one-dimensional operators. In the region bounded by nodes with the indices $2 \le j \le N_j - 1$ and $2 \le k \le N_1$, the scheme had the form

$$q^{n+1}_{j,k} = P_{\xi}\left(\frac{\Delta t}{2}\right) P_{\eta}\left(\frac{\Delta t}{2}\right) P_{\eta}\left(\frac{\Delta t}{2}\right) P_{\xi}\left(\frac{\Delta t}{2}\right) q^{n}_{j,k}\,.$$

Each one-dimensional operator included the predictor step and the corrector step. Thus, for example, the action of the operator $P_{\xi}(\Delta/2)$ on the column vector $q^{n}_{j,k}$ as a result of which the transition to an intermediate value $\overline{q_{j,k}}$ was carried out involved the realization of two steps:

$$q^{0}_{j,k} = q^{n}_{j,k} - \frac{\Delta t}{\Delta \xi}\,[F^{n}_{j+1,k} - F^{n}_{j,k}] + \frac{\Delta t}{4}\,H^{n}_{j,k}\,;\quad \overline{q_{j,k}} = 0.5\,(q^{n}_{j,k} + q^{0}_{j,k}) - 0.5\,\frac{\Delta t}{2\Delta \xi}\,[F^{0}_{j,k} - F^{0}_{j-1,k}] + 0.5\,\frac{\Delta t}{4}\,H^{0}_{j,k}\,.$$

In the region of bunching of the grid nodes $2 \le j \le N_j - 1$ and $N_1 \le k \le N_k - 1$, the scheme of splitting included $2n$ one-dimensional operators P_{η}:

$$q_{j,k}^{n+1} = P_\xi\left(\frac{\Delta t}{2}\right)P_\eta\left(\frac{\Delta t}{2n!}\right)\dots P_\eta\left(\frac{\Delta t}{2}\right)P_\xi\left(\frac{\Delta t}{2}\right)q_{j,k}^{n},$$

where $n = \Delta y/\Delta y_1$. The symmetric picture of one-dimensional operators is required for preservation of the second order of accuracy of the numerical method [4].

To suppress oscillations on shock waves, we used the acoustic scheme of correction of fluxes of Lax–Wendroff [4] consisting of six steps:

(1) calculation of diffusion fluxes: $f_{j+1/2,k}^{\mathrm{d}} = \nu_{j+1/2,k}^{\mathrm{d}}(q_{j+1,k}^{n} - q_{j,k}^{n})$;
(2) calculation of antidiffusion fluxes: $f_{j+1/2,k}^{\mathrm{ad}} = \nu_{j+1/2,k}^{\mathrm{ad}}(q_{j+1,k}^{*} - q_{j,k}^{*})$;
(3) calculation of the diffusion of the solution: $q_{j,k}^{**} = q_{j,k}^{*} + f_{j+1/2,k}^{\mathrm{d}} - f_{j-1/2,k}^{\mathrm{d}}$;
(4) calculation of the first differences: $\Delta q_{j,k}^{**} = q_{j+1,k}^{**} - q_{j,k}^{**}$;
(5) restriction of antidiffusion fluxes

$$S = \operatorname{sign}\left(f_{j+1/2,k}^{\mathrm{ad}}\right);\quad f_{j+1/2,k}^{\mathrm{cad}} = S\max\left[0, \min\left(S\,\Delta q_{j-1/2,k}^{**},\ \left|f_{j+1/2,k}^{\mathrm{ad}}\right|, S\,\Delta q_{j+3/2,k}^{**}\right)\right];$$

(6) construction of the antidiffusion solution: $q_{j,k}^{n+1} = q_{j,k}^{**} - f_{j+1/2,k}^{\mathrm{cad}} - f_{j-1/2,k}^{\mathrm{cad}}$.

At steps (1) and (2) of the correction scheme, the coefficients of diffusion $\nu_{j+1/2,k}^{\mathrm{d}}$ and antidiffusion $\nu_{j+1/2,k}^{\mathrm{ad}}$ depend on the coordinates

$$\nu_{j+1/2,k}^{\mathrm{d}} = \eta_0 + \eta_1\left(u_{j+1/2,k}^{n}\frac{\Delta t}{\Delta x}\right)^2;\quad \nu_{j+1/2,k}^{\mathrm{ad}} = \eta_0 + \eta_2\left(u_{j+1/2,k}^{*}\frac{\Delta t}{\Delta x}\right)^2.$$

Here $u_{j+1/2,k} = 0.5(u_{j,k} + u_{j+1,k})$; $\Delta x = x_{j+1,k} - x_{j,k}$; $q_{j,k}^{n}$ is the solution of system (2) on the time layer t^n, while $q_{j,k}^{*}$ is the intermediate solution obtained upon employment of the explicit scheme of McCormack with time splitting. As the coefficients η_0, η_1, and η_2, we took the minimum values for which no oscillation solutions on shock waves occurred.

The intense vibrations of the gas in the tube are turbulent in character [7]. In order to take into account the turbulent loss, we used an algebraic model of turbulent viscosity. The turbulent viscosity inside the boundary layer for $n < 0.7u_{\mathrm{e}}$, where u_{e} is the velocity in the flow core, was determined in terms of the mixing length [4]:

$$\mu_{\mathrm{t}} = \rho l^2\left(\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right)^{1/2};\quad l = 0.41y\,(1 - \exp(-y^{+}/A));\quad y^{+} = u_\tau y\rho/\mu;\quad u_\tau = \left(\frac{\mu}{\rho}\frac{\partial u}{\partial y}\right)^{1/2};\quad A = 26.$$

Here y is the distance from the solid wall in the normal direction. The turbulent viscosity in the exterior of the boundary layer was described using the Clauser formula [4]:

$$\mu_{\mathrm{t}} = 0.0168\rho u_{\mathrm{e}}\delta^{*}I;\quad \delta^{*} = \int_0^\delta\left(1 - \frac{u}{u_{\mathrm{e}}}\right)dy;\quad I = (1 + 5.5\,(y/\delta)^6)^{-1}.$$

On the tube axis ($k = 1$, $2 \le j \le N_j - 1$), we prescribed the boundary conditions of symmetry in the form of conditions of zero extrapolation for all gasdynamic functions: $u(j, 1) = u(j, 2)$, $v(i, 1) = v(j, 2)$, $p(j, 1) = p(j, 2)$, $E(j, 1) = E(j, 2)$, $T(j, 1) = T(j, 2)$, and $\rho(j, 1) = \rho(j, 2)$.

On solid surfaces, including the surface of a moving piston, we prescribed adhesion conditions for the velocity components and zero-extrapolation conditions for the density, the pressure, the energy, and the temperature.

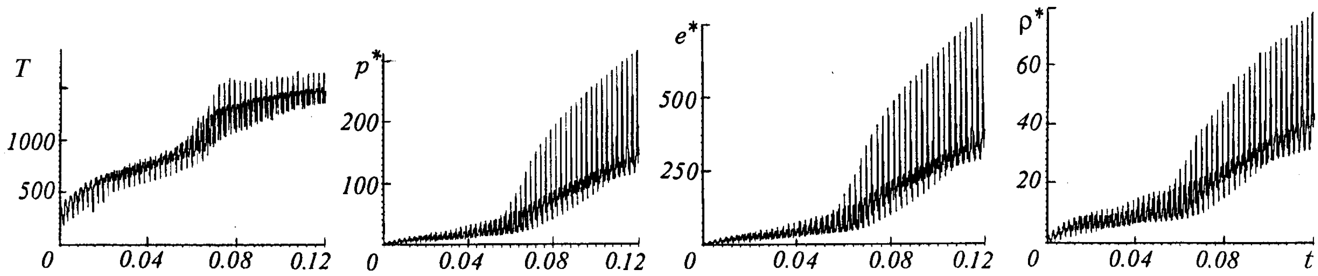

Fig. 2. Time dependence of the gasdynamic functions in excitation at the frequency of the first linear resonance $\omega_{11}(T = 290$ K). T, K; t, sec.

The lateral surface ($k = N_k$, $2 \le j \le N_j - 1$): $u(j, N_k) = 0$, $v(j, N_k) = 0$, $p(j, N_k) = p(j, N_k - 1)$, $E(j, N_k) = E(j, N_k - 1)$, $T(j, N_k) = T(j, N_k - 1)$, and $\rho(j, N_k) = \rho(j, N_k - 1)$.

The closed end of the tube ($j = N_j$, $2 \le k \le N_k - 1$): $u(N_j, k) = 0$, $v(N_j, k) = 0$, $p(N_j, k) = p(N_j - 1, k)$, $E(N_j, k) = E(N_j - 1, k)$, $T(N_j, k) = T(N_j - 1, k)$, and $\rho(N_j, k) = \rho(N_j - 1, k)$.

The piston surface ($j = 1$, $2 \le k \le N_k - 1$): $u(1, k) = \omega a \cos(\omega t)$, $v(1, k) = 0$, $p(1, k) = p(2, k)$, $E(1, k) = E(2, k)$, $T(1, k) = T(2, k)$, and $\rho(1, k) = \rho(2, k)$.

The thermal regime prescribed using the boundary conditions corresponded to a heat-insulated system. At the initial instant of time, we prescribed the temperature, density, and velocity of the gas at the internal nodes of the calculated region. It was assumed that at $t = 0$ the gas at the internal nodes is at rest. The piston moved according to the harmonic law $x(t) = a \sin(\omega t)$; the longitudinal velocities of the gas on the piston surface were equal to $u = a\omega$ at the initial instant of time.

We give below calculation results obtained for the first linear and the second subharmonic resonances for the amplitude of piston vibrations $a = 0.1$ m, the tube length $L = 0.4$ m, and the diameter $d = 0.04$ m. The calculations were performed for the initial temperature of the undisturbed gas $T = 290$ K and the initial density $\rho = 1.2$ kg/m^3. The values of the dynamic viscosity and of the thermal-conductivity coefficient were approximated using a spline of third order based on tabulated data in the temperature interval 290 K $\le T \le$ 1500 K. The results were obtained on a computational grid with the parameters $N_j = 79$, $N_k = 25$, $N_1 = 10$, and $r = 0.8$.

We consider the transformation of the frequency and mode of vibrations of the gas column due to the increase in the gas temperature in excitation at the frequency of the first linear resonance. In this case, the frequency of piston vibrations was $\omega_{11} = \pi a_0(T = 290\ \text{K})/L$, where $a_0(T = 290$ K) is the velocity of sound in the tube for the initial temperature $T = 290$ K. Figure 2 gives the time dependences of the temperature, pressure, internal energy, and frequency of the gas on the axis of symmetry of the tube near the piston. The pressure, internal energy, and density of the gas are indicated in dimensionless quantities: $\rho^* = \rho/\rho_0$; $e^* = e/\rho_0 a_0^2$; $p^* = p/\rho_0 a_0^2$. In the initial stage, we observe one period of vibrations of the gas column during one period of excitation. The frequency and amplitude of gas vibrations are very high, and the temperature in the heat-insulated tube average over the volume increases intensely with time due to the turbulent loss and friction on the walls. The portion of a comparatively slow increase in the average values of the gasdynamic functions $0 < t < 0.065$ sec is disrupted by its sharp increase, which is resonant in character at an instant of time of about $t = 0.065$ sec, as is seen from Fig. 2. The local maximum of the temperature amplitude is observed at $t = 0.065$ sec, and in the vicinity of this point the amplitude decreases against the background of the increasing average value. More detailed consideration of the character of vibrations in the vicinity of resonance (Fig. 3) shows that the time dependences of the gasdynamic functions have a form characteristic of subharmonic vibrations of the ω_{12} type [2]: during one period of piston vibrations, we observe two jumps of the gasdynamic quantities. The resonance burst is observed for the average temperature of the gas in the tube $T \approx 1160$ K (Fig. 2). This temperature approximately coincides with the average temperature on the piston

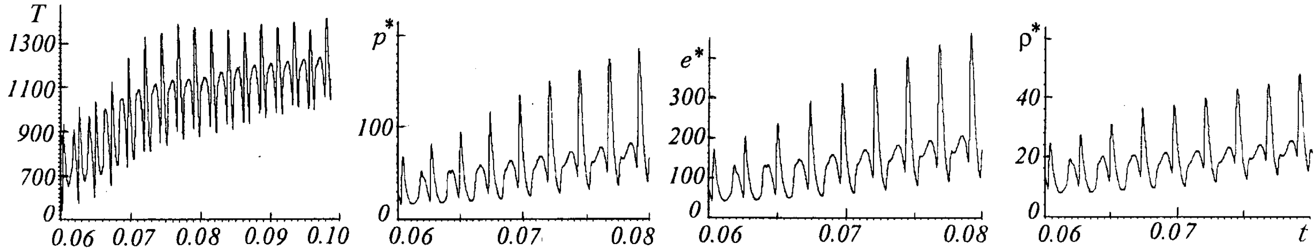

Fig. 3. Modes of vibrations in the vicinity of the resonance $\omega_{12}(T$ = 1160 K).

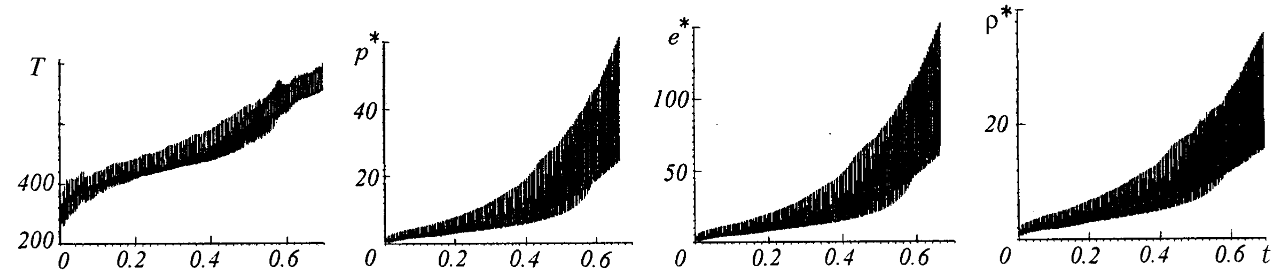

Fig. 4. Time dependence of the gasdynamic functions in excitation at the frequency of the second subharmonic resonance $\omega_{12}(T$ = 290 K).

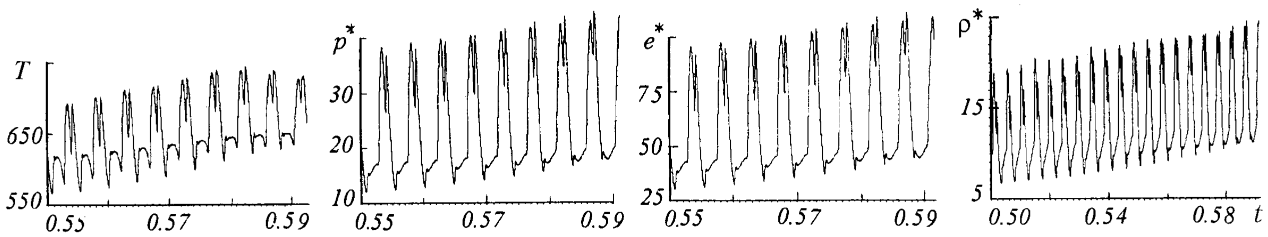

Fig. 5. Modes of vibrations in the vicinity of the resonance $\omega_{13}(T$ = 652 K).

over the period of vibrations near resonance. With a fourfold increase in the temperature (from T = 290 K at t = 0 to T = 1160 K for t = 0.065 sec), the velocity of sound in the system increases twofold, since $a_0 = (\gamma RT)^{0.5}$, where γ is the adiabatic exponent and R is the universal gas constant. The lower eigenfrequency of vibrations of the system $\omega_{11}(T$ = 1160 K) turns out to be twice as high as the fixed frequency of piston vibrations $\omega = \omega_{11}(T$ = 290 K). Such a relation of the frequency of piston vibrations and the eigenfrequency of gas-column vibrations coincides with the conditions of occurrence of the subharmonic resonance ω_{12}, as a result of which the frequency of vibrations of the gas column in the vicinity of the resonance is doubled.

We consider a change in the frequency and mode of vibrations of the gas column due to the increase in the gas temperature in excitation at the frequency of the first subharmonic resonance. In this case, the vibrational frequency of the piston $\omega_{12} = \pi a_0(T = 290\text{ K})/(2L)$ is twice as low as the frequency of the first linear resonance of the gas column, and the gas is heated more slowly. As the gas-temperature average over the tube volume increases 2.25 times as compared to the initial temperature and attains the value T = 652 K by the instant of time $t \approx 0.57$ sec, we observe a resonant burst of the temperature, the pressure, the internal energy, and the density against the background of the increase in the average values (Fig. 4). The velocity of sound and the eigenfrequency of vibrations of the gas column in the tube increase 1.5 times. For a system with the temperature T = 652 K the frequency of piston vibrations corresponds to the frequency of the subharmonic resonance ω_{13}: $\omega_{12}(T = 290\text{ K}) = \pi a_0(T = 290\text{ K})/(2L) = 2\pi a_0(T = 652\text{ K})/(6L) = \pi a_0(T = 652\text{ K})/(3L) = \omega_{13}(T = 652\text{ K})$. As a result, the nonlinear resonance ω_{13} is realized in the system [2, 3]: during one period of vibrations of the piston, we observe three peaks of the gasdynamic functions (Fig. 5) and the vibrational frequency of the gas column increases 1.5 times as compared to the initial frequency. It is noteworthy that the resonance $\omega_{13}(T = 2.25T_0)$ in excitation with the frequency $\omega_{12}(T = T_0)$ is substantially weaker than the resonance $\omega_{12}(T = 4T_0)$ in excitation at the frequency $\omega_{11}(T = T_0)$.

Thus, the calculations performed make it possible to say that the intense vibrations of a gas in a closed tube in the case where the amplitude of piston vibrations is comparable to the tube length lead to a substantially nonstationary process even for a fixed excitation frequency. These vibrations are accompanied by a continuous change of their mode, which is caused by the rapid increase in the lower eigenfrequency of the gas column due to dissipative loss. In excitation of the gas column at the eigenfrequencies $\omega_{11}(T = T_0)$ and $\omega_{12}(T = T_0)$, the transformation of the vibration mode leads to the manifestation of nonlinear resonances of higher frequency in whose vicinity we observe an intense increase in the average and amplitude values of the gasdynamic functions.

This work was carried out within the framework of project No. A0012 of the Federal target-oriented program "Integration" with financial support from the Russian Foundation for Basic Research (project code 99-01-00234) and the Foundation of Research and Development Work of the Republic of Tatarstan.

NOTATION

x and y, longitudinal and radial coordinates in the physical region; ξ, η, corresponding generalized coordinates; u and v, longitudinal and radial components of the gas velocity; t, time; ρ, p, T, E, and e, density, pressure, temperature, and total and internal energy of the gas; ρ_0 and a_0, density and velocity of sound in the undisturbed gas at $t = 0$; μ, molecular viscosity; μ_t, turbulent viscosity; $\mathrm{Pr}_t = 0.9$, turbulent Prandtl number; c_p, specific coefficient of heat capacity; k, thermal-conductivity coefficient; J, Jacobian of transformation in transition from the coordinates x, y, t to coordinates ξ, η, τ; ω_{mn}, resonant frequencies of the gas column; ω, cyclic frequency of vibrations of the piston; L, tube length; d, tube diameter; a, amplitude of vibrations of the piston; l, mixing length in the turbulent-viscosity model; δ, δ^*, and I, thickness of the boundary layer, displacement thickness, and limiting factor; u_e, velocity at the external boundary of the boundary layer; Δ, difference quantity. Subscripts and superscripts: j and k, grid nodes in the direction of the ξ and η axes; t, turbulent; e, external; 0, corresponds to the value of the parameters at the initial instant of time.

REFERENCES

1. M. A. Ilgamov, R. G. Zaripov, R. R. Galiullin, and V. B. Repin, *Appl. Mech. Rev.*, **49**, No. 3, 137–154 (1996).
2. R. G. Zaripov, R. I. Davydov, and N. V. Sonin, *Prikl. Mekh. Tekh. Fiz.*, **40**, No. 6, 61–63 (1999).
3. A. L. Tukmakov and R. G. Zaripov, *Izv. Vyssh. Uchebn. Zaved.*, *Aviats. Tekh.*, No. 1, 64–67 (2001).
4. C. A. J. Fletcher, *Computational Techniques for Fluid Dynamics* [Russian translation], Moscow (1991).
5. V. M. Kovenya, G. A. Tarnavskii, and S. G. Chernyi, *Use of the Method of Splitting in Problems of Aerodynamics* [in Russian], Novosibirsk (1990).
6. J. L. Steger, *AIAA J.*, **16**, No. 7, 679–686 (1978).
7. A. I. Gulyaev and V. M. Kuznetsov, *Inzh. Zh.*, **3**, Issue 2, 236–245 (1963).